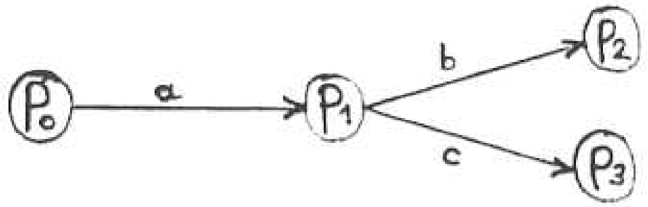
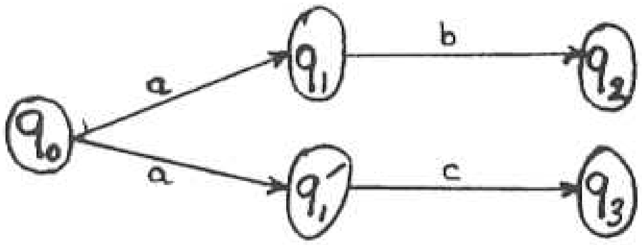
UNDERSTANDING CCS EQUIVALENCES

Consider two vending machines B_1 and B_2 with these STGs:

B_1 :



B_2 :



The behaviours of these two machines cannot be considered equivalent; machine B_1 can simulate machine B_2 but not vice versa. Informally, to say B_1 simulates B_2 will mean that B_1's behaviour pattern is at least as rich as that of B_2.

For a given LTS (Q, Υ), let S be a binary relation over Q. Then S is called a strong simulation over (Q, Υ)

$$\text{if} \quad \begin{array}{ccc} p & S & q \\ \downarrow \alpha & & \\ p' & & \end{array} \quad \text{then for some } q', \quad \begin{array}{ccc} & & q \\ & & \downarrow \alpha \\ p' & S & q' \end{array}$$

For above example, $S = \{(q_0, p_0), (q_1, p_1), (q_1', p_1), (q_2, p_2), (q_3, p_3)\}$ is a strong simulation.

To verify this, for each $(q, p) \in S$ we have to consider each transition $q \xrightarrow{\alpha} q'$ of the first member q, and show that it is properly matched by some transition $p \xrightarrow{\alpha} p$ of the second member p.

REVIEW OF EQUIVALENCE IN CCS

The basic variations of equivalence relations for agents are:

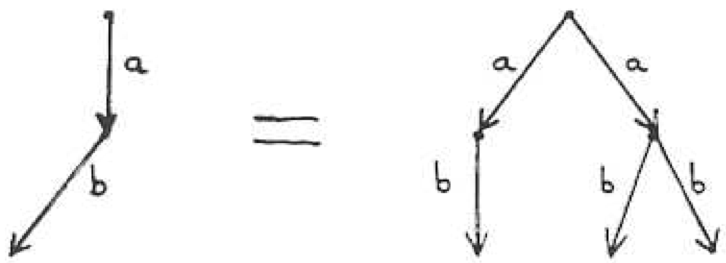
- strong equivalence, in which τ is just as common as any other action, i.e. we distinguish between $\alpha.0$ and $\alpha.\tau.0$. (This is a strict equivalence relation).

- observation equivalence, in which τ is invisible, i.e. it cannot be observed or detected by an external agent. (This is a weak equivalence relation).

The basic idea behind agent equivalence is this: two agents should be strongly equivalent if and only if they have identical derivation trees.

NOTE	A derivation tree is just a state transition graph without the intermediate nodes.

We will stipulate that the agents represented by the following trees are equivalent:



Formally: P and Q are equivalent iff, for every action α, every α-derivative of P is equivalent to some α-derivative of Q and conversely.

NOTE	α-derivative means any action that can follow action α (any branch of the tree below α).

THE CCS EQUIVALENCE RELATIONS

Bisimulation between P and Q intuitively means that P and Q must match each other action for action, and subsequently do whatever each other can do.

NOTE The symbol $\mathcal{P}$ denotes the set of all agents.

STRONG BISIMULATION If $(P,Q) \in \mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$, $\forall \alpha \in Act$,

$$\forall P \xrightarrow{\alpha} P', \exists Q'.\ Q \xrightarrow{\alpha} Q' \wedge (P',Q') \in \mathcal{S}$$
$$\forall Q \xrightarrow{\alpha} Q', \exists P'.\ P \xrightarrow{\alpha} P' \wedge (P',Q') \in \mathcal{S}$$

P and Q are strongly bisimilar/strongly equivalent if $(P,Q) \in \mathcal{S}$ for some strong bisimulation $\mathcal{S}$, written $P \sim Q$.

WEAK BISIMULATION If $(P,Q) \in \mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$, $\forall \alpha \in Act$,

$$\forall P \xrightarrow{\alpha} P', \exists Q'.\ Q \overset{\hat{\alpha}}{\Rightarrow} Q' \wedge P' \approx Q'$$
$$\forall Q \xrightarrow{\alpha} Q', \exists P'.\ P \overset{\hat{\alpha}}{\Rightarrow} P' \wedge P' \approx Q'$$

P and Q are observationally equivalent/weakly bisimilar if $(P,Q) \in \mathcal{S}$ for some weak bisimulation $\mathcal{S}$, written $P \approx Q$.

OBSERVATIONAL CONGRUENCE If $(P,Q) \in \mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$, $\forall \alpha \in Act$,

$$\forall P \xrightarrow{\alpha} P', \exists Q'.\ Q \overset{\alpha}{\Rightarrow} Q' \wedge P' \approx Q'$$
$$\forall Q \xrightarrow{\alpha} Q', \exists P'.\ P \overset{\alpha}{\Rightarrow} P' \wedge P' \approx Q' \text{ ; we write } P = Q.$$

$\forall P, Q.\ P \sim Q \Rightarrow P = Q \Rightarrow P \approx Q$ (from weakest to strongest)